STUDIES ON MIXING. XXVII.*
LIQUID CIRCULATION IN A SYSTEM WITH AXIAL MIXER AND RADIAL BAFFLES**

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This paper deals with the total circulation of liquid mixed by an axial high-speed mixer (propeller mixer or paddle mixer with inclined blades) in a cylindrical vessel with radial baffles. The method of a particle indicating the flow trajectory is used. On the basis of experimentally determined circulation, the flow of the charge in turbulent flow by the use of the total flow rate is quantitatively described, and the effect of geometric parameters on quantities characterizing the liquid flow is studied. At the same conditions it is proved and experimentally verified that a unique relation exists between the mean time of total circulation and the mean time of homogenisation of miscible liquids, independent on the type of the axial mixer as long as Re $> 1{\cdot}0 \,.\, 10^{-4}$.

For different operations taking place in apparatuses with a charge mixed by an rotary high-speed mixer, the mechanical mixing is a factor significantly accelerating the approach to homogeneity of the system *i.e.* equalization of concentration and/or temperature differences. The requirement of elimination of gradients of properties in a charge caused by a step change may be fulfilled if an intensive convective flow takes place in the system.

This paper deals with circulation of the mixed liquid in a cylindrical vessel with radial baffles.

In literature[1-7] quantities are given which enable to characterize the flow in the mixed system as a whole, *i.e.* without considering the detailed local flow characteristics of pressure or velocity vectors at any place. First of all they are: Volumetric or pumping capacity of the mixer $\dot{V}_P$, total volumetric flow rate $\dot{V}_C$ and the induced liquid flow rate $\dot{V}_E$. They are defined:

The *volumetric mixer capacity* or the so called primary flow rate is the flow rate of the liquid from the cylinder by bottom and jacket circumscribed to the rotating mixer (which we shall further call the rotor region).

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The *total volumetric flow rate* of the mixed liquid is the liquid flow rate through the horizontal cross-section of the vessel in the mixer plane in the upward direction per unit of time.*

The *induced flow rate* is that part of the total flow rate in the mixer plane which is not passing through the rotor region. It is caused by the momentum transfer between the primary flow rate and the environment surrounding it.

The cited quantities are part of the dimensionless numbers defined as follows: Criterion of the volumetric or pumping capacity (in literature[3,5,6] it is usually denoted as "flow rate criterion")

$$K_P \equiv \dot{V}_P/nd^3 , \tag{1}$$

criterion of the total flow rate

$$K_C \equiv \dot{V}_C/nd^3 \tag{2}$$

and criterion of the induced flow rate

$$K_E \equiv \dot{V}_E/nd^3 . \tag{3}$$

Obviously the relation is valid

$$K_E = K_C - K_P . \tag{4}$$

Porcelli and Marr[3] suggested a way of determination of these flow characteristics for the case of a charge in a vessel with radial baffles with the axial mixer (propeller) as a source of motion. The method was based on tracing the circulation of the indicating particle. Their considerations were based on four assumptions: The region of the mixed charge is fully filled by the primary and induced flow *i.e.* regions of stagnation do not exist. Between both quoted flows a mass transfer in a macrodimension takes place. The probability that an arbitrary particle of the mixed charge belongs to the primary or induced flow rate is determined by the portion of the volumetric flow rate corresponding to the corresponding partial stream and to the sum of volumetric flow rates of both flows. The accidental residence time of particle carried by any of both streams of given volumetric flow rate has, in the region where the flow takes place, a mean value expressed by ratio of volume of the region considered and of the volumetric flow rate of the given stream.

The authors' consideration[3] led to the definition of two characteristic data:

Mean time of the primary circulation $\bar{\tau}_I$ is the mean time interval between two subsequent passages of the liquid through the rotor region.

Mean time of total circulation $\bar{\tau}_C$ is the average time interval between two subsequent turns of the vertical motion component of the considered particle (or by the indicating particle which is representing it) in the chosen direction *i.e.* always downward or always upward (such change of particle motion will be further called the turn of the particle) without taking into consideration whether the liquid particle has passed through the rotor region or not.

For these times at the above given assumptions holds

$$\bar{\tau}_I = V/\dot{V}_P , \tag{5a}$$

and

$$\bar{\tau}_C = V/\dot{V}_C , \tag{5b}$$

where V is the volume of the mixed charge.

* From this definition it is obvious that the total flow rate of the mixed liquid defined in this way is not very suitable for the description of the flow caused by a mixer of the radial type (turbine mixer[5]) because in this case a flow of liquid through the mixer plane does not need to take place at all (in ideal case if the mixer is placed in the middle between the vessel bottom and the liquid surface liquid may circulate in two regions which are symmetrical to this plane). The total volumetric flow rate would in this case equal to zero even that the circulation in both these regions would be very intensive.

On basis of these considerations of the flow pattern in the studied system as well as on validity of given assumptions a relation was suggested[3] which relates the induced and primary flow rates: the induced flow takes place in the region of a hollow cylinder which extends from the region of a cylinder with a diameter equal to the mixer diameter where the primary flow streams. Therefore the quantities $\dot{V}_E$ and $\dot{V}_P$ may be mutually related by the relation

$$\dot{V}_E = A \frac{D^2 - d^2}{d^2} \dot{V}_P \tag{6}$$

and for the total volumetric flow rate may be written

$$\dot{V}_C = \dot{V}_P \left(1 + A \frac{D^2 - d^2}{d^2}\right). \tag{7}$$

The experiments of the cited authors[3] were made with a rubber sphere which represented a particle of the mixed charge. From the results of these experiments follows, that quantities K_P, K_C, and K_E are functions of geometric simplexes describing the arrangement of the system (D/d, D/h_2, type of baffles) but they are not dependent on hydrodynamic conditions of the system as long as $\text{Re} > 1{\cdot}0 \cdot 10^4$. But the measured data are significantly spread as the number of measurements made has not been sufficient due to the statistical character of the operation. Also the assumptions which lead to relations (*5a*) and (*5b*) were not experimentally verified[3]. Such verification was made by one[6] of authors of this paper who had compared different procedures of measurement of the pumping capacity of the propeller mixer and where he had shown that the measurement of the mean time of primary circulation was suitable for determination of the cited quantity. In this way was also confirmed validity of the model assumptions made by Porcelli and Marr[3]. By use of the cited experimental method the determination of the effect of geometric conditions in the mixed system on the volumetric capacity of the axial mixer was possible. Relations[8,9] were obtained which characterize the pumping effects of basical types of axial mixers: of a three-blade propeller mixer ($s = d$), six-blade paddle mixer with inclined blades ($\alpha = 45°$) and of a three-blade paddle mixer with inclined blades ($\alpha = 24°$) in a system with four radial baffles. The obtained relations between the dimensionless numbers of the exponential type

$$K_P = B(D/d)^e (D/h_2)^f \tag{8}$$

are given in Table I. It was also confirmed that criterion of the pumping capacity is independent on Re number for turbulent regime of the flowing charge ($\text{Re} > 1{\cdot}0 \cdot 10^4$).

TABLE I

Pumping Capacity of Axial High-Speed Mixers for $\text{Re} > 1{\cdot}0 \cdot 10^3$

Mixer type	B	e	f
Propeller ($s = d$)	0·592	−0·146	0·026
Three-blade paddle ($\alpha = 24°$)	0·387	0·130	0·060
Six-blade paddle ($\alpha = 45°$)	1·014	−0·212	0·166

In some papers[10,11] is measured distribution of the vector of the point in time averaged velocity of liquid mixed by a six-blade paddle mixer with inclined blades ($\alpha = 45°$) in a vessel with four baffles. The measurement was made by the use of a three-holed Pitot tube. In this way measurement of the absolute value and of direction of the mentioned vector in the region of two-dimensional flow in the system was possible. The total volumetric flow rates through the given horizontal cross-section of the vessel were calculated from the measured velocity profiles of the liquid mixed by integration over the measured cross-sections. In dimensionless form the exponential dependence of the total flow rate criterion on the ratio D/d was calculated which has the form

$$K_C = K(D/d)^a , \qquad [D/h_2 = \text{const}] . \tag{9}$$

Values of constants of Eq. (9) are given in Table II. All measurements were made at $D/h_2 = 4$ for $\text{Re} > 1{\cdot}0 \,.\, 10^4$.

As one of the aims of this paper is the quantitative comparison of the total circulation caused by the mixer with its homogenisation effects some results which were of use in this paper are mentioned here. The homogenisation of miscible liquids may be studied at dispersion of the added sample of solution having a concentration different from that of the charge (further only "sample") in the mixed system. The homogenisation operation may be quantitatively characterized by the homogenisation time Θ. What concerns the exact definition of this quantity we refer to the paper of Procházka and Landau[13]. Basically it is the time necessary for equalization of the concentration difference in limits of the in advance given deviation from the theoretical value which results from a balance of the considered component.

On the basis of considerations of the homogenisation mechanism several models were published by the use of which the course of homogenisation with time in the mixed system is described when as parameters, are considered the geometric conditions of the system.* Some models of the

TABLE II
Total Volumetric Flow Rate of a Six-Blade Paddle Mixer with Inclined Blades ($\alpha = 45°$)

Data taken from	h_1	K	a
10	0·182D below the mixer plane	0·379	1·010
11	0·060D above the mixer plane	0·686	0·954
11	0·164D above the mixer plane	0·213	1·138
This work	—	0·449	1·129

* Because of the results of the experiments we shall consider in this paper only the turbulent region of the flow regime of the mixed charge when the rate of homogenisation is independent on Re number.

homogenisation course[1,2,12] are based on the assumption that after adding the sample its circulation in the charge takes place or in a dispersed state after passing through the rotor region[2] of the mixer or as an individual particle of the charge[12]. This is accompanied by a diffusion between the particle of the liquid with different concentration from that of the charge. The diffusion (usually of a turbulent character) takes place only in the rotor region[2] or only in the space outside it[12]. As an important quantity is considered the mean time of primary circulation $\bar{\tau}_I$ (see (Eq. 5*a*)) which is characterizing the mean circulation loop in the system. Some other model of homogenisation[13] is based on the assumption that after the addition of the sample into the rotor region this sample is distributed by the turbulent diffusion in the system. The result of these considerations is a semiempirical relation

$$\mathrm{Ho} = K_1 (D/d)^{a_1} \log [2{\cdot}0/C(\Theta)], \; [D/h_2 = \mathrm{const}, \; \mathrm{Re} > 1{\cdot}0 \, . \, 10^4] \,, \tag{10}$$

where

$$\mathrm{H} \equiv n\Theta \tag{11}$$

and where K_1 and a_1 are constants which must be determined experimentally for the given type of the mixer and of the baffles. $C(\Theta)$ is the degree of homogeneity of the system for which as was shown by the cited authors[13] may be substituted the degree of homogeneity in an arbitrary position of the system, defined by

$$C(\Theta) \equiv \sum_{i=1}^{m} \left| \frac{c_i(\Theta) - c_k}{c_0 - c_k} \right| , \tag{12}$$

where m is the number of repeated measurements in the given place of the charge. The authors[13] have evaluated their data according to relation (*10*). From this cited paper the relation for a three-blade propeller mixer ($s = d$) was used and is given in Table III. Kvasnička[14] measured the homogenisation time in the turbulent region of the charge flowing in dependence on geometric simplexes if $C(\Theta) = 0{\cdot}05$ for the six-blade paddle mixer ($\alpha = 45°$) and for the three-blade paddle mixer ($\alpha = 24°$) with inclined blades. In Table III are the results of this paper also recalculated to the general degree of homogeneity of the system according to Eq. (*10*).

TABLE III

Equation for the Rate of Homogenisation of Miscible Liquids

Literature	Mixer type	K_1	a_1	h_2
13	propeller ($s = d$)	3·48	2·05	$D/2$
14	three-blade paddle ($\alpha = 24°$)	4·52[a]	1·95	d
14	six-blade paddle ($\alpha = 45°$)	2·50[a]	2·15	$D/2$

[a] Recalculated according to Eq. (*10*).

THEORETICAL

Relation between the Circulation and Rate of Homogenisation of Miscible Liquids

At the study of the total circulation of a homogeneous mixed charge a particle of liquid is observed represented by an indicating particle whose properties are assumed to be the same as those of the mixed medium. At the study of homogenisation of miscible liquids a sample of liquid of volume ΔV (very small as compared to the volume of the charge, but its dimensions are by several orders larger than the dimensions of molecules) is observed in which the initial concentration c_0' of the solid dissolved substance is different from the initial concentration c_0 in the charge. This sample whose physical properties (viscosity and density) are not significantly different from that of the mixed medium is added into the flowing mixed liquid. As long as the intensity of the convective flow of the mixed liquid is sufficient and as the initial momentum of the added sample is not too large then immediatelly after its addition it moves with the same velocity as the medium which is surrounding it. Thus the sample may be considered as an arbitrary particle of the mixed liquid which is accidentally circulating in the system and its motion may be observed or modelled by the use of the above mentioned indicating particle.

After addition of the sample into the charge mass transfer takes place of the dissolved substance between the sample and the charge. We assume that in this case at steady mixing conditions, inside the sample an unsteady turbulent diffusion of the dissolved substance in liquid takes place, the turbulent diffusivity of which is not changing with time. Inside the sample beside this mechanism also the convective mass transfer takes place. Thus we assume that inside the sample such motion of clusters of molecules of the solution takes place that the gradient of velocity averaged with time of medium inside such volume ΔV has a negligible value.

To be able to study in detail the relation between the total circulation and homogenisation of the dissolved matter we shall consider a coordinate system firmly connected with the sample. Further we shall make several assumptions concerning the sample and the concentration of the dissolved substance introduced with the sample into the system: *1*. The sample in the mixed system has a spherical shape ΔV of a constant diameter. *2*. The motion of the dissolved matter inside the sample may be described on the basis of the conception of turbulent diffusivity (coefficient of turbulent diffusivity in the sample volume is not a function of possition, time and concentration), other effects are negligible. *3*. The distribution of concentrations of the dissolved matter in the sample and on its surface is spherically symmetrical. *4*. At the beginning (*i.e.* at the moment of introduction of the sample into the charge) the concentration of the dissolved matter in the sample as well as in the charge is homogeneous. *5*. At a certain very long time interval after the addition of the sample there will not be any significant difference between the concentration of the dissolved matter in the charge and in the sample. *6.* Rate of mass transfer of the dissolved matter between the sample and the charge which is surrounding it is proportional to the concentration difference between the sample surface and the bulk concentration of the charge surrounding it. The coefficient of mass transfer is not a function of possition, time and concentration.

Let us present several definition relations:

Circulation time is the time interval between two subsequent turns of the sample

$$\tau_C \equiv \int_{M_i}^{M_{i+1}} \mathrm{d}l/v(l)\,,$$

where $v(l)$ is the value module of the velocity vector in the considered place of the path. The given integral is a line integral along the path of the traced sample between two subsequent points M_i and M_{i+1} in which the vertical projection of the path passes through two subsequent local maxima. Mean circulation time defined by Porcelli and Marr[3] may be therefore written as the relation

$$\bar{\tau}_C \equiv (1/N) \sum_{i=1}^{N} \tau_{Ci}\,. \qquad (D1)$$

Degree of homogeneity of the charge is determined by the instantaneous concentration of the dissolved matter (averaged over the volume of the charge, sample exclusive) and by the initial and final concentration of the substance in the charge

$$\begin{aligned} C(t) &\equiv [\overline{c(t)} - c_k]/[c_0 - c_k]\,, \\ \overline{c(t)} &\equiv (1/\mathscr{V}) \int_{\mathscr{V}} c(\mathbf{r}, t)\, \mathrm{d}\mathscr{V}\,, \\ c_0 &\equiv \overline{c(0)}\,, \\ c_k &\equiv \overline{c(\infty)}\,. \end{aligned} \qquad (D2)$$

Degree of homogeneity of the sample is given by the same quantities as in the preceding paragraph and is defined for the sample by

$$\begin{aligned} C'(t) &\equiv [\overline{c'(t)} - c'_k]/[c'_0 - c'_k]\,, \\ \overline{c'(t)} &\equiv (1/\Delta V) \int_{\Delta V} c'(\mathbf{r}, t)\, \mathrm{d}V\,, \\ c'_0 &\equiv \overline{c'(0)}\,, \qquad c'_k \equiv \overline{c'(\infty)}\,. \end{aligned} \qquad (D3)$$

Homogenisation time is the time interval between the moment of introduction of the sample into the charge and the moment when the degree of homogeneity of the sample exceeds the in advance defined value C_h. Thus

$$\Theta \equiv t_h\,, \quad [C(t_h) = C(\Theta) = C_h]\,. \tag{D4}$$

Relations Between the Mean Time of Total Circulation and the Homogenisation time

On the basis of the definition relations (*D2*) and (*D3*) the material balance of the indicating dissolved matter may be written

$$\overline{c(t)}\,\mathscr{V} + \overline{c'(t)}\,\Delta V = c_0\mathscr{V} + c_0'\,\Delta V = c_k\mathscr{V} + c_k'\,\Delta V\,, \tag{13}$$

where the integration regions are defined as follows: ΔV is the region occupied at any moment by the added sample with the invariable volume and shape according to the assumption *1*; $\mathscr{V}$ is the region occupied by the charge exclusive the region ΔV occupied by the sample.

From relation (*13*) we obtain

$$[\overline{c(t)} - c_k]/[c_0 - c_k] = [\overline{c'(t)} - c_k']/[c_0' - c_k']$$

and thus

$$C(t) = C'(t)\,. \tag{14}$$

Thus it holds that the degree of the charge homogeneity equals to the degree of homogeneity of the sample at the same moment. With regard to relation (*D4*) a conclusion is made that the time of homogenisation may be determined on the basis of tracing the concentration changes of the moving sample.

On the basis of assumption *1*, to *3*, concentration changes of the dissolved matter in the sample may be described by equation

$$\frac{\partial[rc'(r,t)]}{\partial t} = D_c\,\frac{\partial^2[rc'(r,t)]}{\partial r^2}\,, \quad [t > 0\,, \quad 0 < r < R]\,, \tag{15}$$

with boundary conditions in the centre and on the surface of the sample with the diameter R, for which

$$\frac{\partial c'(0,t)}{\partial r} = 0\,, \quad c'(0,t) \neq \infty \tag{16a}$$

and

$$D_c \frac{\partial c'(R, t)}{\partial r} + k_c[c'(R, t) - \overline{c(t)}] = 0 \tag{16b}$$

and with the initial condition

$$c'(r, 0) = c'_0 \tag{17}$$

which results from the assumption *4*.

The term $c(t)$ in Eq. (*16b*) may be written in respect to Eq. (*13*) and to the assumption 5. as

$$\overline{c(t)} = c'_k + (\Delta V/\mathscr{V}) [c'_k - c'(t)] .$$

If we substitute this term into Eq. (*16b*) and if we neglect in respect to the assumption of negligence of the sample volume to the charge volume the term with $\Delta V/\mathscr{V}$ as compared to the term $c'(R, t)$, we may define a new variable

$$x(r, t) \equiv c'(r, t) - c'_k \tag{18}$$

and the differential equation (*15*) with initial and boundary conditions may be written in the form

$$\partial[rx(r, t)] = D_c \frac{\partial^2[rx(r, t)]}{\partial r^2}, \quad [t > 0 ; \quad 0 < r < R], \tag{19a}$$

$$\frac{\partial x(0, t)}{\partial r} = 0 ; \quad x(0, t) \neq \infty , \tag{19b}$$

$$D_c \frac{\partial x(R, t)}{\partial r} + k_c[x(R, t)] = 0 , \tag{19c}$$

$$x(r, 0) = c'_0 - c'_k . \tag{19d}$$

The general integral of this system of equations is the term[17]

$$x(r, t) = \sum_{i=1}^{\infty} P_i(1/r) \sin \mu_i(r/R) \exp [-\mu_i^2(D_c/R^2) t] . \tag{20}$$

Values of roots μ_i are obtained by solving the characteristic equation

$$\mathrm{tg}\, \mu_i = [k_c R/D_c) - 1]^{-1} \mu_i . \tag{21}$$

We calculate the mean integral value of function $x(r, t)$ in respect to the sample volume and when we take into consideration Eqs (*14*) and (*18*) we obtain

$$C(t) = \frac{\overline{c'(t)} - c'_k}{c'_0 - c'_k} = \sum_{i=1}^{\infty} Q_i \exp\left(- \mu_i^2 \frac{D_c}{R^2} t\right), \qquad (22a)$$

where

$$Q_i \equiv 6G^2/\mu_i^2(\mu_i^2 + G^2 - G), \qquad (22b)$$

and where*

$$G \equiv k_c R/D_c\,. \qquad (22c)$$

From the system of equations (*22*) for the given degree of homogeneity C_h the homogenisation time $\Theta = t_h$ may be determined.

To be able to verify the dependence between the time of homogenisation and the mean circulation time several steps must be made so that Eq. (*22a*) is simplified. Some of them are based on the empirical experience. At first let us define the quantity

$$J \equiv \Theta/\bar{\tau}_C \qquad (23)$$

as the ratio of the homogenisation time and the mean time of total circulation. Further we shall consider only the first term of the sum on the right side of Eq. (*22a*) which is permissible when $t > 0$ and $G < 1$. The last inequality is fulfilled as is shown in discussion. From geometry of the sample and of the system follows

$$R^2 = \omega D^2\,, \quad \text{where} \quad \omega \sim (\Delta V/\mathscr{V})^{2/3}\,.$$

If Eq. (*22a*) is arranged on the basis of these considerations, we obtain

$$C(\Theta) = Q_1 \exp\left[- \mu_1^2 (D_c/\omega D^2)\, J\bar{\tau}_c\right]. \qquad (24)$$

By the dimensional analysis the relation for the coefficient of turbulent diffusivity may be obtained

$$D_C \sim n(D/d)^{\beta}\,.$$

which is characterizing the transfer of the dissolved substance inside the sample. This dependence was used by Procházka and Landau[13] who by a coefficient of this type have described the mass transfer in the whole volume of the charge.

From relations (*2*), (*5b*) and (*9*) results that $\bar{\tau}_c \sim n(D/d)^{\beta}$. After substitution of these relations into Eq. (*24*) we obtain

$$\Theta/\bar{\tau}_C = J = L(D/d)^{\gamma} \log\left[Q_1/C(\Theta)\right], \qquad (25)$$

* Expression G is the certain analogon of Biot criterion.

from which a conclusion can be made that the homogenisation time is directly proportional to the mean time of circulation while the coefficient of proportionality is a function of the system geometry and of the required degree of homogeneity of the system.

TABLE IV

Data of the Mixed System

a) Physical Properties

Charge	ϱ kg/m^3	η cP
Distilled water	900	0·5
Distilled water	1 000	1·0
Aqueous glycerol	1 084	3·0
Aqueous glycerol	1 143	9·2
Aqueous glycerol	1 161	14·42

b) Geometric Characteristics

D mm	d mm	h_2 mm	Rotational speed of mixer r.p.m.
Propeller ($s = d$) and paddle mixer with three inclined blades ($\alpha = 24°$)			
290	96·6	145·0	450—1 000
290	72·5	96·6	800—1 800
290	58·0	72·5	1 000—2 000
190	58·0	58·0	900—1 600
190	58·0	95·0	900—1 600
190	58·0	58·0	900—1 600
Paddle mixer with six inclined blades ($\alpha = 45°$)			
290	96·6	145·0	300— 600
290	72·5	72·5	600—1 300
290	58·0	72·5	900—1 800
290	46·6	72·5	1 000—2 000
190	58·0	95·0	500—1 000
190	46·0	58·0	500—1 000
190	46·0	46·0	500—1 000

EXPERIMENTAL

The measurements were made in a vessel with a flat bottom and four radial baffles of width b equal to one tenth of the inside vessel diameter (see Fig. 1 and Table IV). The vessel was filled with a homogeneous Newtonian liquid to the height H (with the mixer at rest) above the vessel bottom and this height H was equal to the inside vessel diameter. The measurements were made at 20°C and at 50°C for which reason the vessel was placed in a water bath of a thermostat. The vessel was placed on a metal bench with a vertically movable upper plate. The measurements were made with a three-blade propeller mixer ($s = d$) and with a three-blade paddle mixer with inclined blades with an angle $\alpha = 24°$ and with a six-blade paddle mixer with inclined blades with an angle $\alpha = 45°$ (see Fig. 2*a* and 2*b*). The mixers were rotating in such a direction that the flow streaming from them was oriented downwards. The drive consisted of a 0·5kW direct current electric motor and transmission enabling up to a fourfold reduction of the rotational speed which was kept with an $\pm 1\%$ accuracy by the use of a magnetic controller of the rotational speed Rome. In Table IV are given the basic dimensions of the system used and physical properties of the liquids mixed which were distilled water or water solution of glycerol. At the measurement was visually determined the mean time of total circulation of the indicating particle $\bar{\tau}_C$ in the mixed system. By use of stop-watch was determined the total time τ_C of m_C subsequent turns of the indicating particle, where m_C was registered by the use of an electromagnetic impulse

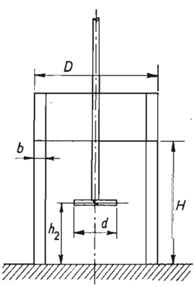

FIG. 1

Schematic View of the Mixed System

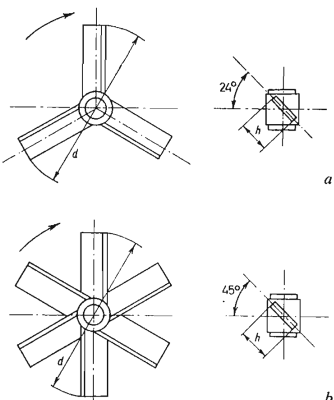

FIG. 2

Mixer

a Three-blade paddle mixer with inclined blades, $\alpha = 24°$, $h = 0{\cdot}2\,d$. *b* Six-blade paddle mixer with inclined blades, $\alpha = 45°$, $h = 0{\cdot}2\,d$.

counter switched on by the operator at each turn. The turn was counted without taking into consideration whether the particle has passed or not. The mean time of total circulation in the rotor region was determined from the relation

$$\bar{\tau}_C = \tau_C / m_C \,. \tag{26}$$

In one series of experiments for the given physical and geometric conditions 1 200 turns of a particle were observed. The series was divided into six parts each with 200 turns. The used indicating particle[15] was of a spherical shape and consisted of three circular plates of 6 mm diameter and of 0·2 mm thickness. It was made of PVC having density 1 110 kg/m^3. The construction of the particle causes that the body is moving in the stream as if it were the liquid in which it occupies a space given by its contour. The resistance of the used shape to the flow around it is large and the difference of its inertia to that of the liquid is small so that the motion indication is satisfactory.

In the described arrangement of the experiments as the independent variables were considered the following quantities: rotational speed of the mixer n, kinematic viscosity of the mixed charge μ/ϱ, mixer diameter d, and the distance of the center of the mixer rotor h_2 from the vessel bottom.

The rotational speed was kept constant with an accuracy of $\pm 1\%$. Temperature of the mixed charge was kept at $20 \pm 1°C$ and at $50 \pm 1°C$ with an accuracy of measurement $\pm 0{\cdot}1°C$. The temperature fluctuation in the given range caused a fluctuation of the kinematic viscosity in the range of $\pm 3\%$. The mixer diameter was measured with a calliper with an accuracy of $\pm 0{\cdot}2$ mm. The distance of the rotor center from the vessel bottom was measured by a steel rule with an accuracy of $\pm$ 0·5 mm.

As the dependent variable was measured the time of total circulation which we consider to be a random quantity. The accuracy of its determination thus depends on the number of repeated measurements and therefore such number of measurements must be made so that the required accuracy be obtained. In several randomly chosen experiments was checked whether the determined number of turns of the indicating particle for the given experiment enabled to reach the required accuracy given by the relative mean square error of the mean time of total circulation. The required value of the mean square error was 3%. In all tested cases better or at least the required accuracy of measurement was obtained.

A certain error at the determination of the time of total circulation is caused by the definition of this quantity which is not quite exact. In the mixed system the region is not exactly limited

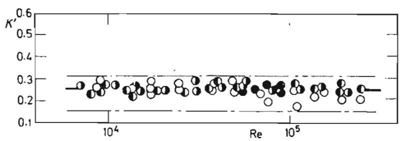

FIG. 3

Resulting Correlation of Total Volumetric Flow Rate of Propeller Mixer ($s = d$)

D/d	3	4	5	3·27
Symbol	○	◐	◑	●

where the indicating particle should be traced as its turns at the motion change are not always of the same quality. This particle moves in a different way after passing through the mixer plane and in a different way if it passes directly through the rotor region. In measurements only those circulations were taken into consideration where the time of total circulation was larger than 0·1 s. In this case it is possible to observe such motion of the particle when its circulation may be still considered and not the rotation alone. Further the minimum rotational speeds were determined for the used mixer types and for the considered ratios D/d at which the particle was not delayed at walls or at the baffles and where the circulation could be still observed. The measurements were made at higher than minimum speeds of rotation of the mixer.

In summary we may state that values of the independent variables were affected by practically negligible errors as compared with errors affecting the determination of the dependent variables.

RESULTS

Results were evaluated statistically[18]. On the basis of the published papers[3] it was assumed that the flow criterion K_C is in the region where our experiments were carried out independent on Re number *i.e.* that it is dependent only on the geometric parameters of the system mixed according to equation

$$K_C = K(D/d)^a (D/h_2)^b . \tag{9a}$$

Relation (*9a*) was linearized by taking its logarithm and values K, a, and b were calculated by the least square method from the experimental data. In Table V are given the results of such correlations *i.e.* values of constants in Eq. (*9a*) and their standard deviations calculated for all the mixers used. By the statistical test (on the basis of the null hypothesis[18]) it was proved, that value of the exponent b in Eq. (*9a*) for the three-bladed paddler and for the propeller mixer is not significant on the 95% confidence level. The calculated correlations were further studied in a graphical way. As the dependent variable was considered the parameter K' calculated from the relation

$$K' = K_C (D/d)^{-a} (D/h_2)^{-b} , \tag{27}$$

where the substituted exponents a, and b were beforehand calculated by the least square method from the measured data. As the independent variables were considered the parameters D/d and D/h_2. The graphical correlation is here illustrated because of lack of space only on the example of the propeller mixer (Fig. 3). Beside values K' are in Fig. 3 plotted as a solid line the statistically calculated values of constant K from Eq. (*9a*) and as a dash and dot line the limits of the reliability interval of this absolute term on the 95% confidence level. The graphical presentation proves the suitability of the used correlation and it is also obvious from that, that the assumption of independence of K' values on Re number with respect to the accuracy of the obtained correlations was correct.

TABLE V
Resulting Correlations of the Total Volumetric Flow Rate

Mixer type	K	a	b	σ_a
Propeller ($s = d$)	0·248	1·303	0·036	0·0602
Six-blade paddle ($\alpha = 45°$)	0·350	1·129	0·172	0·0406
Three-blade paddle ($\alpha = 24°$)	0·280	1·227	0·036	0·0366
Three-blade paddle ($\alpha = 24°$) ($h_2 = d$)	0·242	1·295	—	0·0604

Relation between the Geometric Parameters and the Convective Flow Intensity of the Liquid Mixed

The induced flow rate is defined by the use of dimensionless numbers in relation (*4*). By the use of this equation, values of criterion K_E were calculated from values of criteria K_P and K_C for the given Reynolds number and for the geometric conditions of the mixed system. On the basis of relation (*6*), the parameter $\bar{A}$ (arithmetic mean value from all calculated cases) was then calculated. All the calculated values were in the limits of the interval of the 95% confidence level.

Equation for the total flow rate in the mixer plane presented earlier[3] has the general form of Eq. (7). By substituting into Eq. (7) for the corresponding values of constant $\bar{A}$, relations may be obtained for the total flow rate caused by the considered mixer types. The comparison of these resulting relations is made in Table VI.

DISCUSSION

Total Volumetric Flow Rate

It is obvious from Table V that the largest total volumetric flow rate is caused for the given ratio D/d by the six-blade paddle mixer with inclined blades ($\alpha = 45°$). The three-blade propeller mixer ($s = d$) gives the total volumetric flow rate of the charge practically the same as the three-blade paddle mixer with inclined blades ($\alpha = 24°$). Kvasnička[14] in his paper compared the power input of the mentioned mixer types. He found out that power inputs of these mixers are the same. At determination of the volumetric capacity of axial mixers[3] it was stated, that values of the volumetric capacity of the three-blade paddle mixer ($\alpha = 24°$) and of the propeller mixer ($s = d$) did not differ significantly at otherwise same conditions.

On the basis of all of these knowledge it may be concluded that the mentioned types of axial mixers have practically the same parameters which characterize their power inputs and pumping effects.

TABLE V
(*Continued*)

σ_b	σ_{K+}	σ_{K-}	N	Re
0·0383	0·0308	0·0486	60	$7{\cdot}23 \,.\, 10^3 - 245{\cdot}5 \,.\, 10^3$
0·0314	0·0606	0·0515	76	$6{\cdot}0 \,.\, 10^3 - 203{\cdot}0 \,.\, 10^3$
0·0213	0·0352	0·0313	59	$6{\cdot}90 \,.\, 10^3 - 285{\cdot}0 \,.\, 10^3$
—	0·0553	0·0464	30	$6{\cdot}0 \,.\, 10^3 - 288{\cdot}0 \,.\, 10^3$

For determination of the total volumetric flow rate of the mixed liquid it was also studied whether the obtained correlation relations were valid for geometrically similar systems. Therefore measurements were made with vessels of two dimensions. Values K' calculated from the results of measurements in both geometrically similar systems are in the reliability interval of K values for the 95% confidence level. From this it may be concluded that the obtained correlation relations may be applied to geometrically similar systems if $\mathrm{Re} > 1{\cdot}0 \,.\, 10^4$.

In Table II are given values of constants of Eq. (9) for the total volumetric flow rate in the given horizontal cross-section of the vessel caused by the six-blade paddle mixer with inclined blades ($\alpha = 45°$) for $D/h_2 = 4$. These results are taken from the cited papers[10,11] and from this work. The given constants are characterizing the total volumetric flow rate in different cross-sections of the vessel in the mixer plane and below the mixer. If we consider the results with the pressure probe as the reference one *i.e.* the presented method of determination of the total flow rate in the given cross-section of the vessel as the direct method, then on basis of results in Table II a conclusion may be made if the quantity $\dot{V}_C$ calculated on the basis of the model assumptions[3] (respective the value K_C in the dimensionless form) is the real total volumetric flow rate in the plane of the mixer. Values of the exponents with respect to their dispersion may be considered identical (which was also proved by a statistical

TABLE VI
Comparison of Relations for the Total Volumetric Flow Rate from Measurements of Total Charge Circulation

Mixer type	K_C/K_P
Propeller ($s = d$)	$1 + 0{\cdot}140\,(D^2 - d^2)/d^2$
Three-blade paddle ($\alpha = 24°$)	$1 + 0{\cdot}132\,(D^2 - d^2)/d^2$
Six-blade paddle ($\alpha = 45°$)	$1 + 0{\cdot}081\,(D^2 - d^2)/d^2$

test on the 95% confidence level, by comparison of the expected values of two selected sets of data[18]). The constants K are then characterizing the magnitute of the total flow rate in different cross-sections of the vessel. From this table it is also obvious that the largest total flow rate is probably in the mixer plane* and that with larger distance from this plane in the vertical direction upward or downward this quantity decreases. Value of the total flow rate calculated from the over-all circulation of the charge may not be ascribed to the mixer plane but to the cross-section in the vessel at certain vertical distance from this plane as the constant K determined from the data obtained by measurements of the total circulation is significantly smaller than the value of this constant determined from data obtained from measurements of the velocity profiles and extrapolated to the mixer plane. We may, therefore, consider the total flow rate calculated from the data of total circulation to be only proportional to the total flow rate in the mixer plane, while the effect of geometric conditions on the value of the discussed quantity may be evaluated from the results of measurements of the total circulation.

Induced Flow Rate

It was mentioned in the literature survey that determinations of the total volumetric capacity of the axial mixer from results of measurements of the primary circulation are credible. As follows from the considerations made at the end of the last paragraph, the flow rate of the induced flow determined from the total circulation by the above mentioned procedure is not the flow rate in the mixer plane but it is only proportional to this quantity. Porcelli and Marr[3] have determined for the three-blade propeller mixer ($s = d$) that values of the parameter A defined by Eq. (6) are in the interval $\langle 0{\cdot}130; 0{\cdot}170 \rangle$. It may be seen in Table VI that our results presented in this paper are in a fair agreement with the above mentioned results. It is interesting that values of constants $\bar{A}$ in the given equation for the propeller ($s = d$) and three-blade paddle ($\alpha = 24°$) mixers are very close. It is possible to assume that differences in values of these constants are insignificant which was proved on the 95% confidence level by a statistical comparison of expected values of two selected sets of data[18]. This results supports the above mentioned conclusions concerning the identical flow parameters for systems with the mentioned mixer types.

Value of the constant $\bar{A}$ for the six-blade paddle mixer with inclined blades ($\alpha = 45°$) significantly differs from the corresponding constants for the propeller ($s = d$) and three-blade paddle mixer with inclined blades ($\alpha = 24°$). This may be caused by the fact that not all the mixers used may be considered as axial *i.e.* that the flow streaming from the blades is parallel with the mixer axis. The six-blade paddle mixer with inclined blades ($\alpha = 45°$) must be considered to be a mixer with

* The mixer plane is considered the horizontal plane passing through the axis of the mixer rotor and dividing the mixer height into half.

the so called combined flow for which the liquid leaves the blades of the mixer with a significant radial velocity component. With the three-blade paddle mixer used in this work the radial flow at the exit from its blades practically does not take place as the angle of inclination of blades to the horizontal plane is only 24°. This mixer has practically the same effects for causing the convective flow of the charge as the propeller mixer. From these facts follows that the model assumptions concerning the induced flow may be used as well with the mixers causing the combined flow. From the measured results also follows that the portion of the pumping capacity on the total flow rate is for mixers with the combined flow significantly larger than for purely axial mixers.

From the experiments results it follows that with respect to the dispersion of the obtained correlation the assumption was confirmed, that the constant A is independent on the Re number. This result confirms the conclusions of authors[3] that the constant A is independent of the hydrodynamic regime of the charge if $\mathrm{Re} > 1 \cdot 0 . 10^4$. The total flow rate is the sum of the primary and of the induced flow rate. By use of Eq. (7) this quantity may be calculated on the basis of the known pumping mixer capacity and of the geometric parameters of the system when only one parameter – the constant $\bar{A}$ – is known. The volumetric mixer capacity is a quantity exactly defined and is easily measurable[6].

The Total Circulation of the Liquid Mixed and Homogenisation of Miscible Liquids

As follows from the considerations made in the theoretical part of this paper, the quantity J *i.e.* the ratio of the mean circulation time and the homogenisation time is not a function of the rotational speed (for turbulent regime of mixing) as long as the quantity G is less than 1. By the cited paper[13] the values of D_c were in the range of orders $10^{-4}-10^{-2}$ m^2/s and the order of quantity R was about[16] 10^{-2} m. The order of the mass transfer coefficient was estimated on the basis of several relations which were the result of dimensional analysis and which were given in the monography of Štěrbáček and Tausk[19] in the range of $10^{-2}-10^{-4}$ m/s. From the same equations is obtained that $k_c \sim (nd)^a$ where the exponent $a = 1$. By a simple calculation the validity of the assumption concerning behaviour of the quantity G may be confirmed.

The resulting Eq. (25) was derived for the concentration change of the dissolved matter averaged over the whole charge volume which enables one to overlook the stochastic character of the operation which would appear in case we would study the local concentration change.

Procházka and Landau[13] have experimentally proved that the averaged concentration change may be studied on the basis of local changes in an arbitrary place of the mixed charge as long as we eliminate by a large number of measurements made the stochastic character of the operation (the quoted approximation is obviously not valid for small values of t, *i.e.* when $t \rightarrow 0_+$). Therefore, the value of Q_l experimentally determined by authors[13] is different from the theoretical one, *i.e.* for $G \rightarrow 0$ it holds that $Q_l \rightarrow 1$.

Therefore, we have considered it possible to correlate mutually the homogenisation times measured by these authors with the mean time of total circulation on the basis of relation (*25*). The results of correlation are given in Table VII*a* for the value $Q_l = 2$. From this Table is apparent the good agreement of the corresponding constants for all types of mixers which was proved by the knowledge of their dispersion by a statistical comparison of the expected values of two selected sets of data[18]. The obtained results quantitatively prove the assumptions concerning the unique relation between the mean time of circulation and the homogenisation time in a system with an axial mixer if the flow character may be considered to be turbulent and if the convective flow interferes with the whole system. The homogenisation time is then a multiple of the total circulation time and increases with the increase of the ratio D/d and with the decrease of the required homogenisation degree of the system. The time of homogenisation of miscible liquids may be determined on the basis of the total circulation time of the charge (which is a quantity much easier to measure) of the geometrical parameters of the system and of the required degree of homogenisation.

In the literature survey of this paper are discussed the models of the homogenisation mechanism[2,12] which are based on the conception that the mass transfer in the mixed system is caused mostly by the primary flow (by the pumping mixer capacity). Therefore, we have correlated on the basis of Eq. (*25*) the homogenisation time and the time of primary circulation. As is obvious from Table VII*b*, the value of constant L significantly differs with different mixer types. Therefore it depends on the type of the mixer used, which is a difference from Table VII*a*. The time of total circulation therefore characterizes more suitably the convective flow in the entire system and its use is thus more suitable for the description of homogenisation than is the mean primary circulation time.

It is interesting that the formally similar type of equation like Eq. (*24*) may be obtained from relation (*3-11*) of the cited paper[2]

$$\bar{c}(\Theta) = (\Delta c/2T)\left[1 - \exp\left(-2\Theta/T\right)\right],$$

when we at first consider that initial concentration of the charge c_0 does not equal to zero (which is the difference from cited paper) and when the time constant T is put equal to the mean time of total circulation $\bar{\tau}_C$ and not to the mean time of primary circulation $\bar{\tau}_I$, as was done by one author[2], than for the degree of homogeneity we obtain

$$\frac{\overline{c(\Theta)} - c_k}{c_0 - c_k} = C(\Theta) = \exp\left(-2\Theta/\bar{\tau}_C\right), \tag{28}$$

TABLE VII

Dependence of Homogenisation Time on Mean Time

Mixer type	L	σ_L	γ	σ_γ
a) of Total Circulation				
Propeller ($s = d$)	1·11	0·253	0·35	0·12
Three-blade paddle ($\alpha = 24°$)	1·39	0·202	0·28	0·12
Six-blade paddle ($\alpha = 45°$)	1·12	0·221	0·25	0·08
b) of Primary circulation				
Propeller ($s = d$)	1·30	0·20	−1·10	0·08
Three-blade paddle ($\alpha = 24°$)	2·03	0·22	−0·86	0·09
Six-blade paddle ($\alpha = 45°$)	0·689	0·20	−1·06	0·16

so that

$$J = \Theta/\bar{\tau}_C = 1{\cdot}15 \log \left[1/C(\Theta)\right], \qquad (29)$$

which is the expression equivalent to that of Eq. (*25*) for value of the constant Q_1 equal to one.

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LIST OF SYMBOLS

A	constant in Eqs (*6*) and (*7*)
a	exponent in Eqs (*9*) and (*9a*)
a_1	exponent in Eq. (*10*)
B	constant in Eq. (*8*)
b	exponent in Eq. (*10a*)
b	width of baffle (m)
$c(\Theta)$	concentration of the dissolved matter in the charge (kg m^{-3})
$c(t)$	concentration of the dissolved matter in the charge the sample region exclusive (kg m^{-3})
$c'(t)$	concentration of the dissolved matter in the sample (kg m^{-3})
Δc	concentration change in the charge caused by the sample addition (kg m^{-3})
D	vessel diameter (m)
D_c	coefficient of turbulent diffusivity (m^2 s^{-1})
d	mixer diameter (m)
e	exponent in Eq. (*8*)
f	exponent in Eq. (*8*)
G	dimensionless number defined by Eq. (*22*)
H	height of liquid surface above the vessel bottom when at rest (m)
h	blade width of the mixer (m)

h_1	distance of the Pitot tube from the mixer plane (m)
h_2	distance of the mixer rotor centre (mixer plane) above the vessel bottom (m)
K	constant in Eqs (*9*) and (*9a*)
K_1	constant in Eq. (*10*)
K'	parameter in Eq. (*27*)
k_c	mass transfer coefficient ($m\,s^{-1}$)
L	constant in Eq. (*25*)
dl	element of path of the indicating particle (m)
M_i	local maximum of the particle path (m)
m	number of repeated measurements in one series of measurements
N	number of circulations of the indicating particle
N	number of pairs (triple) variables in the regression dependence
n	rotational speed of the mixer (s^{-1})
P	integration constant in Eq. (*2*) ($kg\,m^{-2}$)
Q	integration constant in Eq. (*22*)
R	sample diameter (m)
$\mathbf{r}$	radius vector (m)
r	radial coordinate (m)
s	pitch of propeller mixer (m)
V	volume of the charge mixed (m^3)
ΔV	volume of the added sample (m^3)
$\mathscr{V}$	volume of charge without the sample volume (m^3)
v	velocity of the indicating particle ($m\,s^{-1}$)
$\dot{V}$	volumetric flow rate ($m^3\,s^{-1}$)
t	time (s)
t_h	experiment time corresponding to the in advance chosen degree of homogeneity (s)
x	variable defined by Eq. (*18*) ($kg\,m^{-3}$)
α	angle of inclination of blade (deg)
β	exponent of geometric simplex
γ	exponent in Eq. (*25*)
δ	exponent of the geometric simplex
η	dynamic viscosity of the charge ($kg\,m^{-1}\,s^{-1}$)
Θ	homogenisation time (s)
σ_a	standard deviation
σ_{K+}, σ_{K-}	standard deviation of constant K for the possitive and negative deviation of the arithmetic mean
μ	root of the characteristic equation (*21*)
ω	coefficient relating the dimensions of the sample and of the charge
ϱ	charge density ($kg\,m^{-3}$)
τ	circulation time (s)

Dimensionless numbers

$C(t) \equiv \dfrac{\overline{c(t)} - c_k}{c_0 - c_k}$ degree of charge homogeneity

$C'(t) \equiv \dfrac{\overline{c'(t)} - c'_k}{c'_0 - c'_k}$ degree of sample homogeneity

$Ho \equiv n\Theta$ criteria of homochronicity (Strouhal)

$J \equiv \Theta/\bar{\tau}_C$ ratio of homogenisation time and of the total circulation time
$K_C \equiv \dot{V}_C/nd^3$ criteria of the total flow rate
$K_E \equiv \dot{V}_E/nd^3$ criteria of the induced flow rate
$K_P \equiv \dot{V}_P/nd^3$ criteria of the volumetric (pumping) capacity
$\mathrm{Re} \equiv nd^2\varrho/\eta$ Reynolds number

Subscripts and Superscripts

C related to the total circulation or to the total flow rate of the charge
E related to the induced flow rate of the charge
h related to the chosen value of the charge homogeneity
I related to primary circulation
i indice of addition
k final value
o initial value
P related to the volumetric mixer capacity
— mean value over volume
′ related to the sample

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